

New resonance-polariton Bose-quasiparticles enhances optical transmission into nanoholes in metal films.

Minasyan V.N. and Samoilov V.N.

Scientific Center of Applied Research, JINR,
Dubna, 141980, Russia

January 12, 2013

Abstract

We argue the existence of fundamental particles in nature, neutral Light-Particles with spin 1, and rest mass $m = 1.8 \cdot 10^{-4} m_e$, in addition to electrons, neutrons and protons. We call these particles Light Bosons because they create the electromagnetic field which represents Planck's gas of massless photons together with a gas of Light Particles in the condensate. In this respect, the condensed Light Particles, having no magnetic field, represent the constant electric field. In this context, we predict a existence of plasmon-polariton and resonance-polariton Bose-quasiparticles with effective masses $m_l \approx 10^{-6} m_e$ and $m_r = 0.5 m_e$, which are induced by interaction of the plasmon field and the resonance Frölich- Schafroth charged bosons with electromagnetic wave in metal. Also, we prove that the enhancement optical transmission into nanoholes in metal films and Surface Enhanced Raman Spectroscopy are provided by a new resonance-polariton Bose-quasiparticles but not model of surface plasmon-polariton. In this letter, the quantization Fresnel's equations is presented which confirms that Light Particles in the condensate are concentrated near on the wall of grooves in metallic grating and, in turn, represent as the constant electric field which provides the launching of the surface Frölich-Schafroth bosons on the surface metal holes.

1. Introduction.

In the letter [1], we investigated the quantization scheme for the local electromagnetic field which in turn concludes the existence of light quasi-particles with spin 1 and finite effective mass $m = 2.5 \cdot 10^{-5} m_e$ (where m_e is the mass of electron). We showed that the Bose-gas consisting of the light quasi-particles in homogenous medium can be described by the Bose- gas of two sort polaritons. This picture is like to investigation of the superfluid liquid ^4He which was based on the London model [2] where connection between the ideal Bose gas and superfluidity in liquid ^4He was proposed. The ideal Bose gas undergoes a phase transition at sufficiently low temperatures to a condition in which the zero-momentum quantum state is occupied by a finite fraction of the atoms. Later, Landau described the properties of superfluid ^4He in terms of collective excitations so-called phonons and rotons [3]. The purely microscopic theory was first described by Bogoliubov [4] within the model of weakly non-ideal Bose-gas, with the inter-particle S- wave scattering. The Bogoliubov model for superfluid helium have given a description of the gas consisting of atoms helium via Bogoliubov's quasiparticles which reproduces phonons at lower momenta (Landau prediction) as well as free atoms at higher momenta (London's model of ideal gas). As we suggest the picture of describing of superfluidity in liquid ^4He is strongly like to the problem of the model of electromagnetic field. Consequently, there is arising the back problem which is connected with understanding of the essence of electromagnetic wave because we wish to know the origin of particles which may induce the Plank's Bose-gas of massless photons.

For beginning, we shall examine the quantization scheme for local electromagnetic field in the vacuum, as first presented by Planck in his black body radiation studies. It is well known, that classical electrodynamics leads to appearance of the so-called ultraviolet catastrophe; to remove this problem, Planck modeled the electromagnetic field as an ideal Bose gas of massless photons with spin 1. Later, Dirac [5] showed the Planck photon-gas could be obtained through a quantization scheme for the local electromagnetic field, by using of a model Bose-gas of local plane electromagnetic waves.

It is well known, in quantum mechanics, a matter wave is determined by wave-particle duality or de Broglie wave of matter [6] which was confirmed by famous Davisson and Germer experiment, and also by Compton effect [7] where the particle nature of light was demonstrated. This reasoning allows us to present a model of electromagnetic field as the non-ideal Bose-gas consisting of the Bose-particles with spin 1 and having non-zero rest mass. This model is based on the application of the principles of the wave-particle duality and the gauge invariance. As we show this approach allows us to argue the existence of the fundamental neutral Particles with spin 1 and mass $m = 1.8 \cdot 10^{-4} m_e$ in nature, which will be found by experiment. We

call these particles as Light-Particles because they create the gas of massless photons with spin 1.

There have been many studies of optical light transmission through individual nanometer-sized holes in opaque metal films in recent years [8-10]. These experiments showed highly unusual transmission properties of metal films perforated with a periodic array of subwavelength holes, because the electric field is highly localized inside the grooves (around 300-1000 times larger than intensity of incoming optical light). Here we analyze the absorption anomalies for light in the visible to near-infrared range observed into nanoholes in metal films. These absorption anomalies for optical light as it seen as enhanced transmission of optical light in metal films is provided by new resonance effect which is differ from model surface plasmon polaritons (SPP)(collective electron density waves propagating along the surface of the metal films) excited by light incident on the hole array [11].

Hence, we mention about the Raman scattering which is a relatively weak process. The number of photons Raman scattered is quite small. However, as it is known the increase in intensity of the Raman signal for adsorbates on surfaces occurs because of an enhancement in the electric field provided by the surface. Surface Enhanced Raman Spectroscopy, so called SERS, where results in the enhancement of Raman scattering by molecules adsorbed on rough metal surfaces [12,13]. In this context, we show when the incident light strikes the surface of metal, localized new Bose-quasiparticles so called resonance-polaritons are excited. The electric field enhancement is greatest when the frequency of incoming photons is in resonance with the Frölich-Schafroth's bosons. These resonance-polaritons directed along to the surface where they are concentrated.

In this letter, we prove that the absorption anomalies for optical light connected with presence new surface resonance-polaritons but not by SPP as it was accepted. We also demonstrate that new resonance-polariton Bose-quasiparticles in lamellar metallic gratings lead to existence of Light Particles in the condensate, when almost all Light Particles, incoming from air to interface on metal, are concentrated near on the metal-air interface and fall the condensate level. In turn, the Light Particles in the condensate on the metal surface as constant electric field attribute to surface Frölich- Schafroth's bosons for moving along the surface of metal.

11. Light-Particles with spin 1 in vacuum.

For beginning, we consider the Maxwell's equations for electromagnetic field in vacuum [14]:

$$curl\vec{H} - \frac{1}{c} \frac{d\vec{E}}{dt} = 0 \quad (1)$$

$$curl \vec{E} + \frac{1}{c} \frac{d\vec{H}}{dt} = 0 \quad (2)$$

$$div \vec{E} = 0 \quad (3)$$

$$div \vec{H} = 0 \quad (4)$$

where $\vec{H} = \vec{H}(\vec{r}, t)$ and $\vec{E} = \vec{E}(\vec{r}, t)$ are, respectively, the magnetic and electric field vectors depending on space coordinate \vec{r} and time t , and c is the velocity of light in vacuum. Obviously, $\varepsilon = 1$ and $\mu = 1$ are, respectively, the dielectric and the magnetic transmissivity of the vacuum.

The Hamiltonian of the radiation field \hat{H}_R is:

$$\hat{H}_R = \frac{1}{8\pi} \int (E^2 + H^2) dV \quad (5)$$

In order to solve a problem connected with a quantized electromagnetic field in vacuum, we suggest that the electromagnetic field in vacuum consists of N Light-Particles with spin 1 and rest mass m contained into box with volume V . Due to application the principle of wave-particle duality, we may suggest that these particles have the vectors of the electric $\vec{E}_0 = \vec{E}_0(\vec{r}, t)$ and magnetic $\vec{H}_0 = \vec{H}_0(\vec{r}, t)$ fields depending on space coordinate \vec{r} and time t , and turn satisfy to the Maxwell equations:

$$curl \vec{H}_0 - \frac{1}{c} \frac{\partial \vec{E}_0}{\partial t} = 0 \quad (6)$$

$$curl \vec{E}_0 + \frac{1}{c} \frac{\partial \vec{H}_0}{\partial t} = 0 \quad (7)$$

$$div \vec{E}_0 = 0 \quad (8)$$

$$div \vec{H}_0 = 0 \quad (9)$$

To find the relationship between the vectors \vec{E} , \vec{H} and \vec{E}_0 , \vec{H}_0 , we introduce the following expressions which in turn is provided the principle of the gauge invariance:

$$\vec{E} = \alpha curl \vec{E}_0 + \beta \cdot \vec{E}_0 \quad (10)$$

and

$$\vec{H} = \alpha curl \vec{H}_0 + \beta \vec{H}_0 \quad (11)$$

where α and β are the unknown constants.

Thus, the electric \vec{E} and magnetic \vec{H} vectors of initial electromagnetic field are determined by the secondary electromagnetic field with electric \vec{E}_0 and magnetic \vec{H}_0 vectors of the Light-Particles. Obviously, \vec{E}_0 and \vec{H}_0 besides having relationship presented by (6) and (7), they satisfy to the wave-equations:

$$\nabla^2 \vec{E}_0 - \frac{1}{c^2} \frac{\partial^2 \vec{E}_0}{\partial t^2} = 0 \quad (12)$$

$$\nabla^2 \vec{H}_0 - \frac{1}{c^2} \frac{\partial^2 \vec{H}_0}{\partial t^2} = 0 \quad (13)$$

Since, we intend to present the quantized forms for electric and magnetic operator-vectors $\vec{E}_0(\vec{r}, t)$ and $\vec{H}_0(\vec{r}, t)$ of the Light-Particles which are propagated by direction of wave normal \vec{s} , we postulate that the directions of vector-operators of quantization electric and magnetic fields of Light-Particles, and direction of propagation \vec{s} are not changed, at introducing quantization scheme. Now, we express the operator-vector $\vec{E}_0(\vec{r}, t)$ electric field via the second quantization vector wave functions of Light Boson:

$$\vec{E}_0(\vec{r}, t) = A \left(\vec{\psi}(\vec{r}, t) + \vec{\psi}^+(\vec{r}, t) \right) \quad (14)$$

where A is the norm coefficient; $\vec{\psi}(\vec{r}, t)$ and $\vec{\psi}^+(\vec{r}, t)$ are, respectively, the second quantization wave vector functions for one Light-Particle in point of coordinate \vec{r} and time t . These vector functions are creation and annihilation operators of the electric field of one Light-Particle in space coordinate-time, which is directed toward to unit vector \vec{e} perpendicularly to wave normal \vec{s} :

$$\vec{\psi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}} e^{i(k\vec{s}\vec{r} + \omega_{\vec{k}}t)} \quad (15)$$

$$\vec{\psi}^+(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}}^+ e^{-i(k\vec{s}\vec{r} + \omega_{\vec{k}}t)} \quad (16)$$

and

$$\int \vec{\psi}^+(\vec{r}, t) \vec{\psi}(\vec{r}, t) dV = \hat{N}_0 + \sum_{\vec{k} \neq 0} \vec{a}_{\vec{k}}^+ \vec{a}_{\vec{k}} = \hat{N} \quad (17)$$

at

$$\frac{1}{V} \int e^{i\vec{k} \cdot \vec{r}} dV = \delta_{\vec{k}}$$

where $\vec{a}_{\vec{k}}^+$ and $\vec{a}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation of quantized electric field of one Light-Particle in vacuum with wave vector \vec{k} which is directed along wave normal \vec{s} or $\vec{k} = k\vec{s}$. These

Bose vector-operators are directed to the direction of the unit vector \vec{e} which is perpendicular to wave normal \vec{s} ; \hat{N} is the operator total number of light bosons in vacuum; $\hat{N}_0 = \hat{a}_0^\dagger \hat{a}_0$ is the total number of light particles in the condensate.

While investigating superfluid liquid, Bogoliubov [4] separated the atoms of helium in the condensate from those atoms filling states above the condensate. In an analogous manner, we may consider the vector operators $\hat{a}_0 = \vec{e}\sqrt{N_0}$ and $\hat{a}_0^\dagger = \vec{e}\sqrt{N_0}$ as c-numbers within the approximation of a macroscopic number of Light-Particles in the condensate $N_0 \gg 1$. This assumptions lead to a broken Bose-symmetry law for Light-Particles in the condensate. Taking the given conclusion, we get

$$\vec{E}_0(\vec{r}, t) = \vec{E}_{0,0} + \frac{A}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{a}_{\vec{k}} e^{i(k\vec{s}\vec{r} + kct)} + \vec{a}_{\vec{k}}^\dagger e^{-i(k\vec{s}\vec{r} + kct)} \right) \quad (18)$$

where $\vec{E}_{0,0} = 2A\vec{e}\sqrt{\frac{N_0}{V}}$.

Application (18) into (7), by taking into account (13), leads to

$$\vec{H}_0(\vec{r}, t) = \vec{H}_{0,0} - \frac{A}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{s} \times \vec{a}_{\vec{k}} e^{kct} - \vec{s} \times \vec{a}_{-\vec{k}}^\dagger e^{-ikct} \right) e^{i\vec{k}\vec{r}} \quad (19)$$

where

$$\vec{H}_{0,0} = -A\vec{s} \times \vec{e} \left(\sqrt{\frac{N_0}{V}} - \sqrt{\frac{N_0}{V}} \right) = 0$$

In fact, the Light-Particles in the condensate reproduce only the constant electric field $\vec{E}_{0,0} = 2A\vec{e}\sqrt{\frac{N_0}{V}}$ which is directed along unit vector \vec{e} .

In this context, there is an important condition for transverse electromagnetic field $\vec{E}_0 \cdot \vec{H}_0 = 0$ which is easy to prove by using (18) and (19), and equality $\vec{a}(\vec{b} \times \vec{c}) = \vec{c}(\vec{a} \times \vec{b}) = \vec{b}(\vec{c} \times \vec{a})$ and $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

Further calculation claims to find the operators $\frac{\partial \vec{E}_0(\vec{r}, t)}{\partial t}$ and $\frac{\partial \vec{H}_0(\vec{r}, t)}{\partial t}$ which by prescription of Dirac [5], at current time $t = 0$, they take the forms:

$$\frac{\partial \vec{E}_0}{\partial t} = \frac{icA}{\sqrt{V}} \sum_{\vec{k}} k \left(\vec{a}_{\vec{k}} - \vec{a}_{-\vec{k}}^\dagger \right) e^{i\vec{k}\vec{r}} \quad (20)$$

$$\frac{\partial \vec{H}_0}{\partial t} = \frac{icA}{\sqrt{V}} \sum_{\vec{k}} k \vec{s} \times \left(\vec{a}_{\vec{k}} + \vec{a}_{-\vec{k}}^\dagger \right) e^{i\vec{k}\vec{r}} \quad (21)$$

With these new terms \vec{E}_0 and \vec{H}_0 by using of (6) and (7) into (10) and (11), the radiation Hamiltonian \hat{H}_R in (5) takes the form:

$$\begin{aligned}
\hat{H}_R &= \frac{1}{8\pi} \int (E^2 + H^2) dV = \\
&= \frac{1}{8\pi} \int \left[\left(-\frac{\alpha}{c} \frac{\partial \vec{H}_0}{\partial t} + \beta \vec{E}_0 \right)^2 + \right. \\
&\quad \left. + \left(\frac{\alpha}{c} \frac{\partial \vec{E}_0}{\partial t} + \beta \vec{H}_0 \right)^2 \right] dV
\end{aligned} \tag{22}$$

Now, we introduce an expression

$$(\vec{a} \times \vec{b})(\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$$

and a condition for transverse electromagnetic field $\vec{s} \cdot \vec{a}_k = 0$ and $\vec{s} \cdot \vec{a}_k^+ = 0$, as result of application (18) into (8), which helps us to obtain the reduced form of \hat{H}_R in (22). Hence, it is a necessary to indicate that expansion of the vectors $\vec{E}_0(\vec{r}, t)$ and $\vec{H}_0(\vec{r}, t)$ by wave vector \vec{k} is correct, if they are given by different wave vectors \vec{k} and \vec{k}' , respectively. However, the finally result requests an accepting $\vec{k} = \vec{k}'$. Thus

$$\hat{H}_R = \hat{H}_a + \hat{H}_b \tag{23}$$

where the operators \hat{H}_a and \hat{H}_b are:

$$\hat{H}_a = \frac{1}{8\pi} \int \left[\frac{\alpha^2}{c^2} \left(\frac{\partial \vec{H}_0}{\partial t} \right)^2 - \frac{2\alpha\beta}{c} \frac{\partial \vec{H}_0}{\partial t} \vec{E}_0 + \beta^2 \vec{E}_0^2 \right] dV \tag{24}$$

and

$$\hat{H}_b = \frac{1}{8\pi} \int \left[\frac{\alpha^2}{c^2} \left(\frac{\partial \vec{E}_0}{\partial t} \right)^2 + \frac{2\alpha\beta}{c} \frac{\partial \vec{E}_0}{\partial t} \vec{H}_0 + \beta^2 \vec{H}_0^2 \right] dV \tag{25}$$

with the terms involving above equations which are

$$\frac{1}{8\pi} \int \frac{\alpha^2}{c^2} \left(\frac{\partial \vec{H}_0}{\partial t} \right)^2 dV = \sum_{\vec{k}'} \frac{\alpha^2 A^2 k'^2}{8\pi} \left(\vec{a}_{\vec{k}'} + \vec{a}_{-\vec{k}'}^+ \right) \left(\vec{a}_{-\vec{k}'} + \vec{a}_{\vec{k}'}^+ \right) \tag{26}$$

$$\frac{1}{8\pi} \int \frac{\alpha^2}{c^2} \left(\frac{\partial \vec{E}_0}{\partial t} \right)^2 dV = - \sum_{\vec{k}} \frac{\alpha^2 A^2 k^2}{8\pi} \left(\vec{a}_{\vec{k}} - \vec{a}_{-\vec{k}}^+ \right) \left(\vec{a}_{-\vec{k}} - \vec{a}_{\vec{k}}^+ \right) \tag{27}$$

$$\frac{1}{8\pi} \int \beta^2 \vec{E}_0^2 dV = \sum_{\vec{k}} \frac{\beta^2}{8\pi} \left(\vec{a}_{\vec{k}} + \vec{a}_{-\vec{k}}^+ \right) \left(\vec{a}_{-\vec{k}} + \vec{a}_{\vec{k}}^+ \right) \tag{28}$$

$$\frac{1}{8\pi} \int \beta^2 \vec{H}_0^2 dV = \sum_{\vec{k}'} \frac{\beta^2}{8\pi} \left(\vec{a}_{\vec{k}'} - \vec{a}_{-\vec{k}'}^+ \right) \left(\vec{a}_{-\vec{k}'} - \vec{a}_{\vec{k}'}^+ \right) \tag{29}$$

$$\frac{2\alpha\beta}{c} \frac{d\vec{H}_0}{dt} \vec{E}_0 = \frac{2\alpha\beta}{c} \frac{d\vec{E}_0}{dt} \vec{H}_0 = 0 \tag{30}$$

If we suggest that $\alpha = \frac{\hbar\sqrt{\pi}}{A\sqrt{2m}}$, then, the Eq.(23) may rewrite down by approximation of a macroscopic number of Light- Particles in the condensate $N_0 \gg 1$, as

$$\begin{aligned}\hat{H}_R = & \frac{\beta^2 A^2 N_0}{2\pi} + \sum_{\vec{k} \neq 0} \left[\left(\frac{\hbar^2 k^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right) \vec{a}_{\vec{k}}^+ \vec{a}_{\vec{k}} - \right. \\ & - \frac{1}{2} \left(\frac{\hbar^2 k^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right) \left(\vec{a}_{\vec{k}}^+ \vec{a}_{-\vec{k}}^+ + \vec{a}_{-\vec{k}} \vec{a}_{\vec{k}} \right) \Big] + \\ & + \sum_{\vec{k}' \neq 0} \left[\left(\frac{\hbar^2 k'^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right) \vec{a}_{\vec{k}'}^+ \vec{a}_{\vec{k}'} + \right. \\ & + \frac{1}{2} \left(\frac{\hbar^2 k'^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right) \left(\vec{a}_{\vec{k}'}^+ \vec{a}_{-\vec{k}'}^+ + \vec{a}_{-\vec{k}'} \vec{a}_{\vec{k}'} \right) \Big]\end{aligned}\quad (31)$$

We now apply new transformation for vector-operator [1], which is similar to Bogoliubov's one [4], for evaluation the energy levels of the operator \hat{H}_R within diagonal form:

$$\vec{a}_{\vec{k}} = \frac{\vec{i}_{\vec{k}} + L_{\vec{k}} \vec{i}_{-\vec{k}}^+}{\sqrt{1 - L_{\vec{k}}^2}} \quad (32)$$

where $L_{\vec{k}}$ is the real symmetrical functions of a wave vector \vec{k} .

The operator Hamiltonian \hat{H}_R by using of a canonical transformation takes a following form:

$$\hat{H}_R = \frac{\beta^2 A^2 N_0}{2\pi} + \sum_{\vec{k}} \chi_{\vec{k}} \vec{i}_{\vec{k}}^+ \vec{i}_{\vec{k}} + \sum_{\vec{k}'} \chi_{\vec{k}'} \vec{i}_{\vec{k}'}^+ \vec{i}_{\vec{k}'} \quad (33)$$

Hence, we infer that the Bose-operators $\vec{i}_{\vec{k}}^+$, $\vec{i}_{\vec{k}'}^+$ and $\vec{i}_{\vec{k}}$, $\vec{i}_{\vec{k}'}$ are, respectively, the vector "creation" and "annihilation" operators of massless photons with energies $\chi_{\vec{k}}$ and $\chi_{\vec{k}'}$:

$$\begin{aligned}\chi_{\vec{k}} &= \sqrt{\left(\frac{\hbar^2 k^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right)^2 - \left(\frac{\hbar^2 k^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right)^2} = \\ &= \frac{\hbar k \beta A}{\sqrt{2m\pi}} = \hbar k c\end{aligned}\quad (34)$$

and

$$\begin{aligned}\chi_{\vec{k}'} &= \sqrt{\left(\frac{\hbar^2 k'^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right)^2 - \left(\frac{\hbar^2 k'^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right)^2} = \\ &= \frac{\hbar k' \beta A}{\sqrt{2m\pi}} = \hbar k' c\end{aligned}\quad (35)$$

where the constant $\beta = \frac{c\sqrt{2m\pi}}{A}$ is defined.

Taking $\vec{k} = \vec{k}'$, we obtain $\chi_{\vec{k}} = \chi_{\vec{k}'}$, and

$$\hat{H}_R = mc^2 N_0 + 2 \sum_{\vec{k} \neq 0} \chi_{\vec{k}} \vec{t}_{\vec{k}}^+ \vec{t}_{\vec{k}} \quad (36)$$

where $mc^2 N_0$ is new term, in regard to Plank's formulae, which determines the energy of Light-Particles in the condensate. This reasoning implies that the Light-Particles in the condensate represent as the constant electric field without magnetic one.

Hence, we note that the operator \hat{H}_R in (31) may present as

$$\hat{H}_R = \hat{H}_e + \hat{H}_h \quad (37)$$

where

$$\hat{H}_e = \sum_{\vec{k}} \left(\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k}}^+ \vec{a}_{\vec{k}} + \frac{1}{2} \sum_{\vec{k}} U_{\vec{k}} \left(\vec{a}_{\vec{k}}^+ \vec{a}_{-\vec{k}}^+ + \vec{a}_{-\vec{k}} \vec{a}_{\vec{k}} \right) \quad (38)$$

and

$$\hat{H}_h = \sum_{\vec{k}} \left(\frac{\hbar^2 k'^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k}'}^+ \vec{a}_{\vec{k}'} + \frac{1}{2} \sum_{\vec{k}'} U_{\vec{k}'} \left(\vec{a}_{\vec{k}'}^+ \vec{a}_{-\vec{k}'}^+ + \vec{a}_{-\vec{k}'} \vec{a}_{\vec{k}'} \right) \quad (39)$$

where

$$U_{\vec{k}} = -\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \quad (40)$$

$$U_{\vec{k}'} = \frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} \quad (41)$$

are the potential interaction between the Light-Particles in wave vector space.

We argue that there is a condition for wave numbers of Light-Particles $k < k_0$ which provides the property of hard particles for Light Particle. This condition requests that the potentials $U_{\vec{k}}$ and $U_{\vec{k}'}$ interaction between particles were a repulsive, which must occur namely for particles which having the form of hard sphere form (recall S-wave repulsive pseudopotential interaction between atoms in the superfluid liquid ^4He in the model hard spheres [15]). However, when

$$U_{\vec{k}} = -\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} > 0 \quad (42)$$

then

$$U_{\vec{k}'} = \frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} < 0 \quad (43)$$

These equations determine the boundary wave number

$$k_0 = \frac{mc}{\hbar} \quad (44)$$

for Light-Particles which separates the particles by repulsive and attractive potentials interaction because for Light Particles, with wave vectors belonging to $0 \leq k < k_0$, are separated by the repulsive potential $U_{\vec{k}}$ and by attractive $U_{\vec{k}'}$ which, respectively, determine but for Light Particles with wave number determines the Light Particles with $U_{\vec{k}} > 0$ as Particles with form of hard sphere, and the Light Particles with $U_{\vec{k}'} < 0$ as Quasiparticles because they may not satisfy to condition of model hard sphere. The separation of the Light Particles on Particles and Quasiparticles which respectively correspond to Particles of electric field and Quasiparticles to magnetic field is confirmed by reasoning that electric field consists of the Light Bosons in the condensate (18) but magnetic field consists of Light Quasiparticles which cannot set into the condensate (19).

Thus, in statistical equilibrium or thermodynamic limit, the average energy of system is presented as

$$\overline{\hat{H}}_R = mc^2 N_0 + 2 \sum_{0 \leq k < k_0} \chi_{\vec{k}} \overrightarrow{i}_{\vec{k}}^+ \overrightarrow{i}_{\vec{k}} \quad (45)$$

where $\overrightarrow{i}_{\vec{k}}^+ \overrightarrow{i}_{\vec{k}}$ is the average number of photons with the wave vector \vec{k} at temperature T :

$$\overrightarrow{i}_{\vec{k}}^+ \overrightarrow{i}_{\vec{k}} = \frac{1}{e^{\frac{\hbar k c}{kT}} - 1} \quad (46)$$

Now, we prove that the existence of Light Particles satisfies to the relativistic theory of Einstein. Considering Light-Particle as de Broglie wave, we may express the boundary wave number k_0 by following form:

$$k_0 = \frac{mv_{max}}{\hbar \sqrt{1 - \frac{v_{max}^2}{c^2}}} \quad (47)$$

where v_{max} is the maximal value speed of the Light-Particle. Substituting quantity boundary wave number k_0 from (45) into (47), we obtain the following equation

$$\frac{mc}{\hbar} = \frac{mv_{max}}{\hbar \sqrt{1 - \frac{v_{max}^2}{c^2}}} \quad (48)$$

which determines a quantity $v_{max} = \frac{c}{\sqrt{2}} < c$. In this context, there is a Mistaken into the paper [16] where it was stated that Relativistic theory,

proposed by Einstein, is incorrect. Just, we showed that the definition of object, as Light-Particles, within model hard sphere, leads to Right Sound of the theory of Einstein, because the maximal value of speed of the Light-Particle is less then the velocity of electromagnetic wave in vacuum $v_{max} = \frac{c}{\sqrt{2}} < c$.

Now, it is a necessary to find the mass of Light Boson which it will be calculated in section IX as experimental result.

111. The gauge invariance.

Thus, we may recall that solutions of (1)-(4) and (6)-(9) have the forms:

$$\vec{E} = -\frac{\partial \vec{A}}{c\partial t} - grad\phi \quad (49)$$

$$\vec{H} = curl\vec{A} \quad (50)$$

and

$$\vec{E}_0 = -\frac{\partial \vec{A}_0}{c\partial t} - grad\phi_0 \quad (51)$$

$$\vec{H}_0 = curl\vec{A}_0 \quad (52)$$

where \vec{A} and ϕ are, respectively, the vector and scalar potentials of initial electromagnetic field; \vec{A}_0 and ϕ_0 are, respectively, the vector and scalar potentials of secondary electromagnetic field or electromagnetic field of Light Bosons.

On other hand, there are presence two conditions of transversely polarized excitations:

$$div\vec{A} = 0 \quad (53)$$

and

$$div\vec{A}_0 = 0 \quad (54)$$

In empty space, far from any electric charges, we usually assume that scalar potentials of initial and secondary electromagnetic fields are zero $\phi = 0$ and $\phi_0 = 0$. Then,

$$\vec{E} = -\frac{\partial \vec{A}}{c\partial t} \quad (55)$$

and

$$\vec{E}_0 = -\frac{\partial \vec{A}_0}{c \partial t} \quad (56)$$

Therefore, inserting the vectors \vec{H} , \vec{E} from (50),(55) and \vec{H}_0 , \vec{E}_0 from (52),(56) into (10) and (11), we obtain

$$\vec{A} = \alpha \text{curl} \vec{A}_0 + \beta \vec{A}_0 \quad (57)$$

By using of the operation *div* to the both sides of Eq.(57), and accepting the Coulomb gauge condition for initial electromagnetic field $\text{div} \vec{A} = 0$ in (53), then we get the result $\text{div} \vec{A}_0 = 0$ presented in (54) automatically, because $\text{div}(\text{curl} \vec{A}_0) = 0$. Thus, the existence of Light Bosons is not contradicting to the quantum electrodynamics because the Coulomb gauge condition for secondary electromagnetic field is fulfilled.

1V. Light-Particles with spin 1 in homogenous medium.

For beginning, we consider the Maxwell's equations for electromagnetic field in homogenous medium which are presented by the forms:

$$\text{curl} \vec{H} - \frac{1}{c} \frac{\partial \vec{D}}{\partial t} = 0 \quad (58)$$

$$\text{curl} \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0 \quad (59)$$

$$\text{div} \vec{D} = 0 \quad (60)$$

$$\text{div} \vec{B} = 0 \quad (61)$$

where $\vec{B} = \vec{B}(\vec{r}, t)$ and $\vec{D} = \vec{D}(\vec{r}, t)$ are, respectively, the local magnetic and electric induction depending on space coordinate \vec{r} and time t ; $\vec{H} = \vec{H}(\vec{r}, t)$ and $\vec{E} = \vec{E}(\vec{r}, t)$ are, respectively, the magnetic and electric field vectors, and c is the velocity of light in vacuum. The further equations are

$$\vec{D} = \varepsilon \vec{E} \quad (62)$$

$$\vec{B} = \mu \vec{H} \quad (63)$$

where $\varepsilon = \varepsilon(\vec{r} - \vec{r}'; t - t')$ and $\mu = 1$ are, respectively, the dielectric and the magnetic susceptibilities of the homogenous medium. Hence, we note that dielectric respond ε of the homogenous medium is only a function of the separations $\vec{r} - \vec{r}'$ and $t - t'$ from the external field at \vec{r}' and t' .

In analogy, manner, as it was made in above, the electric \vec{E} , electric induction \vec{D} and magnetic \vec{H} vectors of initial electromagnetic field are determined by the secondary electromagnetic field with electric \vec{E}_0 , electric induction \vec{D}_0 and magnetic \vec{H}_0 vectors of the Light-Particles in homogenous medium by application of the principles of wave-particle duality and gauge invariance, which in turn lead to relationships:

$$\vec{E} = \alpha \text{curl} \vec{E}_0 + \beta \cdot \vec{E}_0 \quad (64)$$

$$\vec{D} = \alpha \text{curl} \vec{D}_0 + \beta \cdot \vec{D}_0 \quad (65)$$

and

$$\vec{H} = \alpha \text{curl} \vec{H}_0 + \beta \vec{H}_0 \quad (66)$$

where $\alpha = \frac{\hbar\sqrt{\pi}}{A\sqrt{2m}}$ and $\beta = \frac{2c\sqrt{2m\pi}}{A}$.

Thus, the Maxwell's equations for secondary electromagnetic field are presented as

$$\text{curl} \vec{H}_0 - \frac{1}{c} \frac{\partial \vec{D}_0}{\partial t} = 0 \quad (67)$$

$$\text{curl} \vec{E}_0 + \frac{1}{c} \frac{\partial \vec{H}_0}{\partial t} = 0 \quad (68)$$

$$\text{div} \vec{D}_0 = 0 \quad (69)$$

$$\text{div} \vec{H}_0 = 0 \quad (70)$$

with

$$\vec{D}_0 = \varepsilon \vec{E}_0 \quad (71)$$

$$\vec{B}_0 = \vec{H}_0 \quad (72)$$

By prescription of Kubo in [17] for electrical conductivity in solids. In analogy manner, we treat the dielectric response $\varepsilon(\vec{r} - \vec{r}', t - t')$ for homogenous medium. In this respect, we determine the equations for electric induction $\vec{D}_0(\vec{r}, t)$ of plane electromagnetic wave, propagating in fixed direction of unit vector \vec{s} in homogeneous medium. In this case, the electric induction $\vec{D}_0(t)$ may depend on $\vec{E}_0(t')$ in precess moment of time but not next one:

$$\vec{D}_0(\vec{r}, t) = \int d^3r' \int_{-\infty}^t dt' \varepsilon(\vec{r} - \vec{r}'; t - t') \vec{E}_0(\vec{r}', t') \quad (73)$$

By application of the Fourier transformations:

$$\vec{D}_0(\vec{r}, t) = \frac{1}{2\pi} \int_0^{+\infty} d\omega_{\vec{k}} e^{-i\omega_{\vec{k}}t} \vec{D}_0(\vec{r}, \omega_{\vec{k}}) \quad (74)$$

$$\vec{D}_0(\vec{r}, \omega_{\vec{k}}) = \int dt e^{-i\omega_{\vec{k}}t} \vec{D}_0(\vec{r}, t) \quad (75)$$

we obtain

$$\vec{D}_0(\vec{r}, \omega) = \int dr' \varepsilon(\vec{r} - \vec{r}'; \omega_{\vec{k}}) \vec{E}_0(\vec{r}', \omega_{\vec{k}}) \quad (76)$$

where

$$\varepsilon(\vec{r} - \vec{r}'; \omega_{\vec{k}}) = \int_0^{\infty} dt \varepsilon(\vec{r} - \vec{r}'; t) e^{i\omega_{\vec{k}}t} \quad (77)$$

Then, making the Fourier transformation by coordinate space, we obtain in space wave number-frequency:

$$\vec{D}_0(\vec{k}, \omega_{\vec{k}}) = \varepsilon(\vec{k}, \omega_{\vec{k}}) \vec{E}_0(\vec{k}, \omega_{\vec{k}}) \quad (78)$$

where

$$\vec{D}_0(\vec{k}, \omega_{\vec{k}}) = \int d\vec{r} e^{-i\vec{k}\vec{r}} \vec{D}_0(\vec{r}, \omega_{\vec{k}}) \quad (79)$$

Thus, we have an important result

$$\vec{D}_0(\vec{r}, t) = \sum_{\vec{k}} \varepsilon(\vec{k}, \omega_{\vec{k}}) \vec{E}_0(\vec{k}, \omega_{\vec{k}}) e^{i(\vec{k}\vec{r} + \omega_{\vec{k}}t)} \quad (80)$$

In this context, we postulate that the electric $\vec{E}_0(\vec{r}, t)$ operator-vectors of the Light-Particles is expressed via the second quantization vector wave functions of Light Boson which determines a creation and an annihilation the Light-Particle in space coordinate-time by following way:

$$\vec{E}_0(\vec{r}, t) = A \left(\phi(\vec{r}, t) + \phi^+(\vec{r}, t) \right) \quad (81)$$

where A is the scalar amplitude of electric field; $\vec{\phi}(\vec{r}, t)$ and $\vec{\phi}^+(\vec{r}, t)$ are, respectively, the second quantization wave vector functions for one Light-Particle in homogenous medium in point of coordinate \vec{r} and current time t :

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{b}_{\vec{k}} e^{i(\vec{k}\vec{r} + \omega_{\vec{k}}t)} \quad (82)$$

$$\vec{\phi}^+(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{b}_{\vec{k}}^{\dagger} e^{-i(\vec{k}\vec{r} + \omega_{\vec{k}}t)} \quad (83)$$

and

$$\int \vec{\phi}^+(\vec{r}, t) \vec{\phi}(\vec{r}, t) dV = \hat{N}_{0,h} + \sum_{\vec{k} \neq 0} \vec{b}_{\vec{k}}^+ \vec{b}_{\vec{k}} = \hat{N}_h \quad (84)$$

where $\vec{b}_{\vec{k}}^+$ and $\vec{b}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation of the electric field for free one Light-Particles with spin 1 in homogeneous medium, described by a vector \vec{k} and frequency $\omega_{\vec{k}}$. The Bose vector-operators are directed by the electric field along unit vector \vec{e} which is perpendicular to vector \vec{k} ; \hat{N}_h is the operator total number of Light bosons in homogenous medium; $\hat{N}_0 = \hat{b}_0^+ \hat{b}_0$ is the total number of Light Particles in homogenous medium which fill the condensate level $\vec{k} = 0$.

In analogy manner, as it was presented in above, we may consider the vector operators $\hat{b}_0 = \vec{e} \sqrt{N_0}$ and $\hat{b}_0^+ = \vec{e} \sqrt{N_{0,h}}$ as c-numbers within the approximation of a macroscopic number of Light-Particles in homogeneous medium $N_{0,h} \gg 1$ which fill the condensate level. This assumptions lead to a broken Bose-symmetry law for Light- Particles in the condensate. Taking the given conclusion, we get

$$\vec{E}_0(\vec{r}, t) = \vec{E}_{0,0} + \frac{A}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{b}_{\vec{k}} e^{i(\vec{k}\vec{r} + \omega_{\vec{k}} t)} + \vec{b}_{\vec{k}}^+ e^{-i(\vec{k}\vec{r} + \omega_{\vec{k}} t)} \right) \quad (85)$$

Then, by application (67), we obtain

$$\vec{D}_0 = \vec{D}_{0,0} + \frac{A}{\sqrt{V}} \sum_{\vec{k} \neq 0} \varepsilon(\vec{k}, \omega_{\vec{k}}) \left(\vec{b}_{\vec{k}} e^{i(\vec{k}\vec{r} + \omega_{\vec{k}} t)} + \vec{b}_{\vec{k}}^+ e^{-i(\vec{k}\vec{r} + \omega_{\vec{k}} t)} \right) \quad (86)$$

where

$$\vec{D}_{0,0} = 2A\vec{e}\varepsilon(\vec{k} = 0, \omega_{\vec{k}=0}) \sqrt{\frac{N_{0,h}}{V}}$$

Obviously, the Eqs.(67)-(72) lead to the wave equation:

$$\nabla^2 \vec{E}_0(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2 \vec{D}_0(\vec{r}, t)}{\partial t^2} = 0 \quad (87)$$

where substituting $\vec{E}_0(\vec{r}, t)$ and $\vec{D}_0(\vec{r}, t)$ from (85) and (86), we may get to the important equation for Light Particles in homogenous medium:

$$\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} = \frac{kc}{\omega_{\vec{k}}} \quad (88)$$

Obviously, at $\varepsilon(\vec{k}, \omega_{\vec{k}}) = 1$, we deal with electromagnetic field with frequency of Light Particle $\omega_{\vec{k}} = kc$ presented into vacuum.

Thus,

$$\vec{H}_0 = \vec{H}_{0,0} - \frac{A}{\sqrt{V}} \sum_{\vec{k} \neq 0} \sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{s} \times \left(\vec{b}_{\vec{k}} e^{i\omega_{\vec{k}} t} - \vec{b}_{-\vec{k}}^+ e^{-i\omega_{\vec{k}} t} \right) e^{i\vec{k}\vec{r}} \quad (89)$$

where

$$\vec{H}_{0,0} = -A\sqrt{\varepsilon(\vec{k}=0, \omega_{\vec{k}=0})}\vec{s} \times \vec{e}\left(\sqrt{\frac{N_{0,h}}{V}} - \sqrt{\frac{N_{0,h}}{V}}\right) = 0$$

Thus, the condensed Light Particles in homogeneous medium reproduce the constant electric field $\vec{E}_{0,0} = 2A\vec{e}\sqrt{\frac{N_{0,h}}{V}}$ in direction \vec{e} but no having a magnetic field.

Now, we use of a prescription of Dirac [5], at current time $t = 0$, the operators $\frac{\partial \vec{D}_0}{\partial t}$ and $\frac{\partial \vec{H}_0}{\partial t}$ take the forms:

$$\frac{\partial \vec{D}_0}{\partial t} = \frac{iAc}{\sqrt{V}} \sum_{\vec{k}} k\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \left(\vec{b}_{\vec{k}} - \vec{b}_{-\vec{k}}^+\right) e^{i\vec{k}\vec{r}} \quad (90)$$

and

$$\frac{\partial \vec{H}_0}{\partial t} = \frac{iAc}{\sqrt{V}} \sum_{\vec{k}} \sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{k} \times \left(\vec{b}_{\vec{k}} + \vec{b}_{-\vec{k}}^+\right) e^{i\vec{k}\vec{r}} \quad (91)$$

The Hamiltonian of the radiation field \hat{H}_R in homogeneous medium is very complex, as yet not fully solved. This is problem because in optics, namely, the intensity of electromagnetic field plays a main role, which is determined via average vector Poynting $\langle \vec{S} \rangle$:

$$\langle \vec{S} \rangle = \frac{c}{4\pi} \int \left(\vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \right) dV \quad (92)$$

The, we may determine the Hamiltonian of the radiation field \hat{H}_R in homogeneous medium by following formulae:

$$\hat{H}_R = \frac{1}{8\pi} \int \left(\varepsilon^{\frac{1}{2}}(\vec{r}, \vec{r}'; t) E^2 + \frac{H^2}{\varepsilon^{\frac{1}{2}}(\vec{r}, \vec{r}'; t)} \right) dV \quad (93)$$

For plane electromagnetic wave propagating in direction of unit vector \vec{s} :

$$\vec{H}(\vec{r}, t) = \sqrt{\varepsilon(\vec{r}, \vec{r}'; t)} \left(\vec{s} \times \vec{E}(\vec{r}, t) \right) \quad (94)$$

and

$$\vec{E}(\vec{r}, t) = -\frac{1}{\sqrt{\varepsilon(\vec{r}, \vec{r}'; t)}} \left(\vec{s} \times \vec{H}(\vec{r}, t) \right) \quad (95)$$

which substituting into (92), in turn we get the result

$$\langle \vec{S} \rangle = 2c\hat{H}_R\vec{s} \quad (96)$$

where wave normal \vec{s} gives a direction propagation of plane wave, and in turn direction of the average vector Poynting $\langle \vec{S} \rangle$.

Thus, we postulate that the Hamiltonian of the radiation field in homogeneous medium for radiation, propagating toward fixed direction \vec{s} , is presented by (93) but not by

$$\hat{H}_{R,h} = \frac{1}{8\pi V} \int \left(\vec{E} \cdot \vec{D} + \mu H^2 \right) dV \quad (97)$$

which is calculated by case of dielectric respond $\varepsilon(\vec{r}, \vec{r}'; t)$ is the constant.

Now, we calculate the radiation Hamiltonian \hat{H}_R in homogeneous medium (93):

$$\begin{aligned} \hat{H}_R &= \frac{1}{8\pi} \int \left(\varepsilon^{\frac{1}{2}}(\vec{r}, t) E^2 + \frac{H^2}{\varepsilon^{\frac{1}{2}}(\vec{r}, t)} \right) dV = \\ &= \frac{1}{8\pi} \int \left[\varepsilon^{\frac{1}{2}}(\vec{r}, t) \left(-\frac{\alpha}{c} \frac{\partial \vec{H}_0(\vec{r}, t)}{\partial t} + \beta \vec{E}_0(\vec{r}, t) \right)^2 + \right. \\ &\quad \left. + \frac{1}{\varepsilon^{\frac{1}{2}}(\vec{r}, t)} \left(\frac{\alpha}{c} \frac{\partial \vec{D}_0(\vec{r}, t)}{\partial t} + \beta \vec{H}_0(\vec{r}, t) \right)^2 \right] dV \end{aligned} \quad (98)$$

After simple calculation, in analogy manner as it was made in above, with using of approximation of a macroscopic number of Light- Particles in the condensate $N_{0,h} \gg 1$, the radiation Hamiltonian \hat{H}_R in (98) may rewrite down as

$$\begin{aligned} \hat{H}_R &= \frac{\varepsilon^{\frac{1}{2}}(\vec{k}=0, \omega_{\vec{k}=0}) \beta^2 A^2 N_{0,h}}{2\pi} + \sum_{0 < k < k_0} \varepsilon(\frac{1}{2}\vec{k}, \omega_{\vec{k}}) \left[\left(\frac{\hbar^2 k^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right) \vec{b}_{\vec{k}}^+ \vec{b}_{\vec{k}} - \right. \\ &\quad \left. - \frac{1}{2} \left(\frac{\hbar^2 k^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right) \left(\vec{b}_{\vec{k}}^+ \vec{b}_{-\vec{k}}^+ + \vec{b}_{-\vec{k}} \vec{b}_{\vec{k}} \right) \right] + \\ &\quad + \sum_{0 < k' < k_0} \varepsilon^{\frac{1}{2}}(\vec{k}', \omega_{\vec{k}'}) \left[\left(\frac{\hbar^2 k'^2}{2m} + \frac{\beta^2 A^2}{4\pi} \right) \vec{b}_{\vec{k}'}^+ \vec{b}_{\vec{k}'} + \right. \\ &\quad \left. + \frac{1}{2} \left(\frac{\hbar^2 k'^2}{2m} - \frac{\beta^2 A^2}{4\pi} \right) \left(\vec{b}_{\vec{k}'}^+ \vec{b}_{-\vec{k}'}^+ + \vec{b}_{-\vec{k}'} \vec{b}_{\vec{k}'} \right) \right] \end{aligned} \quad (99)$$

Obviously, the operator Hamiltonian \hat{H}_R in (99) takes a diagonal form by using of new transformation for vector-operator [1], for evaluation the energy levels of the operator \hat{H}_R :

$$\vec{b}_{\vec{k}} = \frac{\vec{d}_{\vec{k}} + M_{\vec{k}} \vec{d}_{-\vec{k}}^+}{\sqrt{1 - M_{\vec{k}}^2}} \quad (100)$$

where $M_{\vec{k}}$ is the real symmetrical functions of a wave vector \vec{k} .

Then, we find the finally form of \hat{H}_R :

$$\hat{H}_R = \hat{H}_{R,0} + 2 \sum_{0 < k < k_0} E_{\vec{k}} \hat{d}_{\vec{k}}^+ \hat{d}_{\vec{k}} \quad (101)$$

where term

$$\hat{H}_{R,0} = mc^2 N_{0,h} \varepsilon^{\frac{1}{2}}(\vec{k} = 0, \omega_{\vec{k}=0}) \quad (102)$$

is the energy of Light Particles in the condensate. Otherwise, the Light Particles in the condensate represent as the constant electric field, obviously without magnetic one.

Hence, we infer that the Bose-operators $\hat{d}_{\vec{k}}^+$ and $\hat{d}_{\vec{k}}$ are, respectively, the vector creation and annihilation operators of free polaritons with energy

$$E(\vec{k}, \omega_{\vec{k}}) = \hbar k c \varepsilon^{\frac{1}{2}}(\vec{k}, \omega_{\vec{k}}) = \frac{\hbar k^2 c^2}{\omega_{\vec{k}}} \quad (103)$$

where taking account Eq.(88).

Thus, the energy of polariton $E(\vec{k}, \omega_{\vec{k}})$, as function of \vec{k} and $\omega_{\vec{k}}$ simultaneously, is presented by (103). However, it needs to present $E(\vec{k}, \omega_{\vec{k}})$ as function of only a wave number \vec{k} . To solve this problem, we investigate the thermodynamical and optical properties of metal.

V. Thermodynamical and optical properties of metal.

As we had been showed in recent our paper [18], the formation of a free neutron singlet pairs in a superfluid liquid helium-dilute neutron gas mixture which occurs by term, of the interaction between the excitations of the Bose gas and the density modes of the neutron, mediate an attractive interaction via the neutron modes, which in turn leads to a bound state on a spinless neutron pair. After, we investigated the problem of superconductivity [19] presented by Frölich [20], and demonstrated that the Frölich charged spinless electron pair is created in a phonon gas-electron gas mixture by the term of the interaction between the phonon excitations and electron modes by an induced the effective attractive interaction via electron modes. In turn, there is a bound state on singlet electron pair with binding energy

$$E_0 = -\sqrt{\frac{\alpha \hbar^2}{m_e} \left(\frac{n}{V}\right)^{\frac{5}{3}}} + \frac{\alpha n}{V} < 0$$

which provides the formation of the superconducting phase in superconductor by condition for density electrons of metal $\frac{n}{V}$:

$$\frac{n}{V} > \left(\frac{C^2 m_e}{2 M s^2 \hbar^2} \right)^{\frac{3}{2}}$$

At choosing $C \approx 1\text{ev}$ [19]; $M \approx 5 \cdot 10^{-26}\text{kg}$; $s \approx 3 \cdot 10^3 \frac{m}{\text{sec}}$, we estimated that any metal with density of electrons $\frac{n}{V} > 10^{27}\text{m}^{-3}$ can represent as a superconductor. This assumption is able to explain the isotope effect [21]. However, the strong enough lattice-electron interaction can account the Meissner-Ochsenfeld effect [22] which shows that the superconductor in the presence of an applied magnetic field, at temperature below transition temperature superconductor, leads to an expulsion of the field from the superconductor. In this respect, we suggest that the Meissner-Ochsenfeld effect may be defined by BEC of Light Particles in medium which are been by action of the applied magnetic field. This reasoning implies that the Meissner-Ochsenfeld effect is not connected with transition temperature superconductor but it may be defined by transition temperature of the Bose-gas consisting of the Light Particles. Indeed, the condensed Light Particles in homogeneous medium is not having a magnetic field, which follows from (89).

Now, we may remark the most successful attempt made by Schafroth [23], who considered a model superconductor as the charged ideal Bose gas, consisting of charged electron pairs with charge $e_0 = 2e$ and mass $m_0 = 2m_e$. He defined a superconducting phase in the superconductor by using of a density of charged bosons in the condensate. We call these singlet electron pairs by the names of the Frölich and Schafroth because first Schafroth stated the existence these singlet electron pairs into superconductor with charge $e_0 = 2e$ and mass $m_0 = 2m_e$ but Frölich could discover their earlier then it was made by the Cooper. Thus, any metal may consider by the model of free Frölich-Schafroth's charged singlet bosons with charge $e_0 = 2e$ and mass $m_0 = 2m_e$, if the density of electrons of metal satisfies to the condition $\frac{n}{V} > 10^{27}\text{m}^{-3}$. Otherwise, if metal consists an ion lattice and electron gas, then, the free Frölich-Schafroth's charged singlet bosons are formed.

Hence, we note that the Frölich- Schafroth charged bosons are differ from Cupper's pairs [24] which are formed by electrons filling near to the Fermi level. In this respect, we note that theory of superconductivity, presented by Bardeen, Cooper and Schrieffer, and by Bogoliubov (BCSB) [25] based on the existence of the Cupper's pairs. They asserted that the Frölich effective attractive potential between electrons leads to shaping of two electrons with opposite spins to so-called Cooper pairs around Fermi level. However, hence we suggests that their theory contradicts to the principle of identity of particles because the same electrons can not be separated by free electrons into Fermi levels and by interacting electrons around Fermi one, as they state the Paul exclusion principle. However, this principle states that two identical fermions (particles with half-integer spin) may occupy the same quantum state simultaneously. In fact, the Paul principle is not connected with interaction between fermions as it was accepted in the theory of BCSB.

It is well known that the one-particle excitation spectrum in the charged Bose gas was first investigated by Foldy [26] at $T = 0$ and by Bishop [27] at

or near transition temperature T_c . The random-phase approximation (RPA) dielectric response for ideal charged Bose gas, at finite temperatures, was proposed by Hore and Frenkel [28], where was taken the Coulomb interaction between charged particles. On other hand, as we have been mentioned in above, any metal may consider by the model of an ideal Bose gas consisting of free Frölich-Schafroth's charged singlet bosons with charge $e_0 = 2e$ and mass $m_0 = 2m_e$, if the density electrons of metal satisfies to the condition $\frac{n}{V} > 10^{27}m^{-3}$. In accordance this fact, we use of the Schafroth's model of an ideal charged Bose-gas [19] where the density of bosons in the condensate ρ_s depends on temperature T :

$$\rho_s = \rho \left[1 - \left(\frac{T}{T_c} \right)^{\frac{3}{2}} \right] \quad (104)$$

where $\rho = \frac{1}{2} \cdot \frac{n}{V}$ is the density of bosons presented via density of electrons of metal; $\zeta(x)$ is the Riemann zeta function; k is the Boltzman constant; T_c is the transition temperature.

At $\rho_s = 0$, the transition temperature T_c is

$$T_c = \frac{2\pi\hbar^2}{m_0k} \left(\frac{\rho}{\zeta(\frac{3}{2})} \right)^{\frac{2}{3}}$$

For any metal, the density electrons is $\frac{n}{V} \approx 10^{28}m^{-3}$, therefore, at $\rho = 0.5 \approx 10^{28}m^{-3}$, we have $T_c \approx 10^3K$. This result implies that at room temperature $T = 300K$, we may consider any metal as superconductor because $T \leq T_c$.

Otherwise, the clear form of the complex dielectric respond $\hat{\varepsilon}(\vec{k}, \omega)$, as function from temperature T for ideal charged Bose gas [28] which at the temperatures $T \leq T_c$ is presented by the form:

$$\begin{aligned} \hat{\varepsilon}(\vec{k}, \omega_{\vec{k}}) &= 1 - \frac{\omega_p^2}{\omega_{\vec{k}}^2 - \frac{\hbar^2 k^4}{4m_0^2}} \left[1 - \left(\frac{T}{T_c} \right)^{\frac{3}{2}} \right] + \\ &+ \frac{\omega_p^2}{\rho} \cdot \frac{1}{4\pi^2} \cdot \frac{m_0^2}{\hbar^2 k^3} \cdot \frac{2m_0 k T}{\hbar^2} \frac{\pi}{2i} \times \\ &\times \sum_{j=1}^{\infty} \frac{1}{j} \left[\left(1 + \phi(DC_j) \right) e^{D^2 C_j^2} - \left(1 + \phi(BC_j) \right) e^{B^2 C_j^2} \right] \end{aligned} \quad (105)$$

where $D = i \left(\frac{m_0 \omega_{\vec{k}}}{\hbar k} + \frac{1}{2}k \right)$, $B = i \left(\frac{m_0 \omega_{\vec{k}}}{\hbar k} - \frac{1}{2}k \right)$, $C_j = A^{\frac{1}{2}} j^{\frac{1}{2}}$; $A = \frac{\hbar^2}{2m_0 k T}$; $\Phi(x)$ is the error function:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

and $\omega_p^2 = \frac{4\pi e_0^2 \rho}{m_0}$ is the plasma frequency for the charged gas.

After some algebra, they found the following expansions for $\hat{\varepsilon}(\vec{k}, \omega_{\vec{k}})$ in (93) at $T \leq T_c$, which may present by the form [14]:

$$\hat{\varepsilon}(\vec{k}, \omega_{\vec{k}}) = \varepsilon(\vec{k}, \omega_{\vec{k}}) + i \frac{4\pi\sigma(\vec{k}, \omega)}{\omega} \quad (106)$$

where

$$\begin{aligned} \varepsilon(\vec{k}, \omega) = & 1 - \frac{\omega_p^2}{\omega_{\vec{k}}^2 - \frac{\hbar^2 k^4}{4m_0^2}} \left[1 - \left(\frac{T}{T_c} \right)^{\frac{3}{2}} \right] + \\ & + \frac{k^2 k T}{m_0} \cdot \frac{\omega_p^2}{\left(\omega_{\vec{k}}^2 - \frac{\hbar^2 k^4}{4m_0^2} \right)^3} \frac{\zeta(\frac{5}{2})}{\zeta(\frac{3}{2})} \cdot \left(3\omega_{\vec{k}}^2 + \frac{\hbar^2 k^4}{4m_0^2} \right) \left(\frac{T}{T_c} \right)^{\frac{3}{2}} + \dots + \end{aligned} \quad (107)$$

$$\begin{aligned} \frac{4\pi\sigma(\vec{k}, \omega_{\vec{k}})}{\omega_{\vec{k}}} = & \frac{\omega_p^2}{\rho} \cdot \frac{m_0^3 k T}{3\pi \hbar^4 k^3} \left(\sinh \left(\frac{\hbar \omega}{2m_0 k T} \right) \exp \left[- \left[\left(\frac{m_0 \omega}{\hbar k} \right)^2 + \frac{k^2}{4} \right] \frac{\hbar^2}{2m_0 k T} + \right. \right. \\ & \left. \left. + \dots + \right) \right] \end{aligned} \quad (108)$$

hence $\varepsilon(\vec{k}, \omega_{\vec{k}})$ is the dielectric constant; $\sigma(\vec{k}, \omega_{\vec{k}})$ is the conductivity of metal. Now, we introduce the complex refractive index $\hat{n} = \sqrt{\hat{\varepsilon}(\vec{k}, \omega_{\vec{k}})}$ where n is the real refractive index and k is the attenuation index of metal:

$$\hat{n} = n(1 + ik) \quad (109)$$

where

$$n^2 = \frac{1}{2} \left(\sqrt{\varepsilon^2(\vec{k}, \omega_{\vec{k}}) + \frac{16\pi^2 \sigma^2(\vec{k}, \omega)}{\omega^2}} + \varepsilon(\vec{k}, \omega_{\vec{k}}) \right) \quad (110)$$

and

$$n^2 k^2 = \frac{1}{2} \left(\sqrt{\varepsilon^2(\vec{k}, \omega_{\vec{k}}) + \frac{16\pi^2 \sigma^2(\vec{k}, \omega_{\vec{k}})}{\omega_{\vec{k}}^2}} - \varepsilon(\vec{k}, \omega_{\vec{k}}) \right) \quad (111)$$

VI. Polaritons in metal and BEC.

Now, we show that the equations (88) and (105) can present two original types of the BEC. Hence, we consider two interesting cases.

1. At $T = 0$, the dielectric response in Eq.(105) takes the form:

$$\varepsilon(\vec{k}, \omega_{\vec{k}}, T = 0) = 1 - \frac{\omega_p^2}{\omega_{\vec{k}}^2 - \frac{\hbar^2 k^4}{4m_0^2}} \quad (112)$$

In the case of the plasmon excitations $\varepsilon(\vec{k}, \omega_{\vec{k},p}) = 0$, which in turn allows us to find the frequency of plasmon field:

$$\omega_{\vec{k},p} = \sqrt{\omega_p^2 + \frac{\hbar^2 k^4}{4m_0^2}} \quad (113)$$

Obviously, at $\varepsilon(\vec{k}, \omega_{\vec{k},p}) = 0$, the Eq.(88) takes the form:

$$\varepsilon(\vec{k}, \omega_{\vec{k},p}, T = 0) = \frac{k^2 c^2}{\omega_{\vec{k},p}} \quad (114)$$

which is fulfilled at $k \rightarrow 0$, namely, at existence of BEC because all Light Particles fill in the condensate level $\vec{k} = 0$, and reproduce a constant electric field. At approximation $k \rightarrow 0$, the dielectric response around the plasmon frequency $\omega_{\vec{k}} \sim \omega_p$ takes the form

$$\varepsilon(\vec{k}, \omega_{\vec{k}}) = \frac{k^2 c^2}{\omega_p} \quad (115)$$

which is inserting into (103), and, then we obtain the energy polariton-plasmon field $E(\vec{k}, \omega_{\vec{k}}; T = 0)$ as the function of only the wave vector \vec{k} :

$$E(\vec{k}, \omega_{\vec{k}}; T = 0) = \frac{\hbar k^2 c^2}{\omega_p} = \frac{\hbar^2 k^2}{2m_p} \quad (116)$$

This result means that the polariton of plasmon field represents as the Bose-quasiparticle with effective mass

$$m_p = \frac{\hbar \omega_p}{2c^2} \approx 5 \cdot 10^{-6} m_e \quad (117)$$

because for many metals $\omega_p \approx 10^{16} Hz$. Thus, we may state that near the frequency of plasmon field, the electromagnetic wave in the metal induces the plasmon-polariton Bose-quasiparticles.

On other hand, we may show that such plasmon-polaritons are able to be exited into electron gas at absolute zero because at high frequency limit [29], the dielectric constant of electron gas is

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega_k^2} \quad (118)$$

In the case of plasmon excitations $\varepsilon = 0$, and then, the frequency of plasmon-polariton $\omega_{\vec{k}} = \omega_p$, which in turn, determines the energy of plasmon-polariton presented by Eq.(116).

Thus, the Eq.(116) is an universal because it is a presence at describing of the Bose gas and the electron gas where the plasmon frequency ω_p is not changed in the case of the Bose gas as well as the electron gas:

$$\omega_p = \sqrt{\frac{4\pi e_0^2 \rho}{m_0}} = \sqrt{\frac{4\pi e^2 n}{Vm_e}}$$

where $m_0 = 2m_e$, $e_0 = 2e$ and $\rho = \frac{1}{2} \cdot \frac{n}{V}$.

Indeed, the quantity of the plasma frequency depends on the density of electron into metal which may be determined by gas parameter $r_s = \frac{m_e e^2}{\hbar^2 \left(\frac{4\pi n}{3V} \right)^{\frac{1}{3}}}$. This parameter takes a quantity into interval $1.8 \leq r_s \leq 5.5$ for metals [30]. At a quantity of the plasma frequency $\omega_p \approx 10^{16} Hz$, we obtain m_p presented in (117).

2. Now, we consider the case of the resonance effect, when a frequency of incoming photon $\omega_{\vec{k}}$ coincides with the frequency of one Frölich- Schafroth's charged singlet bosons $\frac{\hbar k^2}{2m_0}$. In this case, the complex dielectric respond $\hat{\varepsilon}(\vec{k}, \omega_{\vec{k},r})$, presented by (105), is infinity or $\hat{\varepsilon}(\vec{k}, \omega_{\vec{k},r}) = \infty$, at finite temperatures. Thus, the resonance frequency is

$$\omega_{\vec{k},r} = \frac{\hbar k^2}{2m_0} \quad (119)$$

To find frequency of new resonance-polaritons around resonance frequency $\omega_{\vec{k},r}$, we substitute $\omega_{\vec{k},r}$ into right side of (88) which may determine a dielectric respond for Light Particles $\varepsilon(\vec{k}, \omega_{\vec{k}}; T)$ around $\omega_{\vec{k},r}$:

$$\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}}; T)} = \frac{2m_0 c}{\hbar k} \quad (120)$$

As we see in (121), at $k = 0$ follows that $\hat{\varepsilon}(\vec{k}, \omega) = \infty$, which implies the appearance of an original type BEC, when all Light Particles fill the condensate level $\vec{k} = 0$. On other hand, we may find the energy of resonance-polaritons $E(\vec{k}, \omega_{\vec{k}}; T)$ around $\omega_{\vec{k},r}$ by substituting $\omega_{\vec{k},r}$ from (120) into (103), and then,

$$E(\vec{k}, \omega_{\vec{k}}; T) = 2m_0 c^2 \quad (121)$$

which does not depend on wave vector \vec{k} and temperature T . Thus,

$$\hat{H}_R = \hat{H}_{R,0} + 2 \sum_{0 < k < k_0} E_{\vec{k}} \vec{d}_{\vec{k}}^+ \vec{d}_{\vec{k}} \quad (122)$$

where the energy of Light Particles in the condensate is

$$\hat{H}_{R,0} = \lim_{k \rightarrow 0} \frac{2m_0 m c^3 N_{0,h}}{\hbar k} \rightarrow \infty \quad (123)$$

This result confirms that namely BEC of Light Particles in the condensate in metal may enhance the optical property of Light and reproduce the constant electric field.

Thus, the resonance effect around the resonance frequency $\omega_{\vec{k},r}$ induces the neutral resonance-polariton as Bose-quasiparticles with a constant energy $E(\vec{k}, \omega_{\vec{k}}; T) = 2m_0c^2$ into metal.

VII. Reflection and transmission of plane wave.

Now we treat the quantization Fresnel's equations. If an incident plane wave propagates, by direction of unit vector \vec{s}^i , across the interface between air and metal, the intensity of the wave will be divided between a reflected, by direction of wave normal \vec{s}^r , and a refracted, by direction of unit vector \vec{s}^t , waves. The vectors electric and magnetic fields of incident $\vec{E}_{0,i}(\vec{r}, t)$ and $\vec{H}_{0,i}(\vec{r}, t)$, reflected $\vec{E}_{0,r}(\vec{r}, t)$ and $\vec{H}_{0,r}(\vec{r}, t)$, and refracted $\vec{E}_{0,t}(\vec{r}, t)$ and $\vec{H}_{0,t}(\vec{r}, t)$ Light-Particles by the matrices:

$$\begin{aligned} \vec{E}_{0,i} &= \begin{pmatrix} \vec{E}_x^i \\ \vec{E}_y^i \\ \vec{E}_z^i \end{pmatrix} = \\ &= \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{a}_{\vec{k},||}^+ \cos\theta_i \\ \vec{a}_{\vec{k},\perp}^+ \\ \vec{a}_{\vec{k},||}^+ \sin\theta_i \end{pmatrix} e^{i\tau_i} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{a}_{\vec{k},||}^+ \cos\theta_i \\ \vec{a}_{\vec{k},\perp}^+ \\ \vec{a}_{\vec{k},||}^+ \sin\theta_i \end{pmatrix} e^{-i\tau_i} \end{aligned} \quad (124)$$

$$\begin{aligned} \vec{E}_{0,r} &= \begin{pmatrix} \vec{E}_x^r \\ \vec{E}_y^r \\ \vec{E}_z^r \end{pmatrix} = \\ &= \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{c}_{\vec{k},||}^+ \cos\theta_r \\ \vec{c}_{\vec{k},\perp}^+ \\ \vec{c}_{\vec{k},||}^+ \sin\theta_r \end{pmatrix} e^{i\tau_r} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{c}_{\vec{k},||}^+ \cos\theta_r \\ \vec{c}_{\vec{k},\perp}^+ \\ \vec{c}_{\vec{k},||}^+ \sin\theta_r \end{pmatrix} e^{-i\tau_r} \end{aligned} \quad (125)$$

$$\begin{aligned} \vec{E}_{0,t} &= \begin{pmatrix} \vec{E}_x^t \\ \vec{E}_y^t \\ \vec{E}_z^t \end{pmatrix} = \\ &= \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{b}_{\vec{k},||}^+ \cos\theta_t \\ \vec{b}_{\vec{k},\perp}^+ \\ \vec{b}_{\vec{k},||}^+ \sin\theta_t \end{pmatrix} e^{i\tau_t} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{b}_{\vec{k},||}^+ \cos\theta_t \\ \vec{b}_{\vec{k},\perp}^+ \\ \vec{b}_{\vec{k},||}^+ \sin\theta_t \end{pmatrix} e^{-i\tau_t} \end{aligned} \quad (126)$$

and using (94), we have

$$\begin{aligned}\vec{H}_{0,i} &= \begin{pmatrix} \vec{H}_x^i \\ \vec{H}_y^i \\ \vec{H}_z^i \end{pmatrix} = \\ &= \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{a}_{\vec{k},\perp}^+ \cos\theta_i \\ \vec{a}_{\vec{k},\parallel}^+ \\ \vec{a}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{i\tau_i} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{a}_{\vec{k},\perp}^+ \cos\theta_i \\ \vec{a}_{\vec{k},\parallel}^+ \\ \vec{a}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{-i\tau_i}\end{aligned}\quad (127)$$

$$\begin{aligned}\vec{H}_{0,r} &= \begin{pmatrix} \vec{H}_x^r \\ \vec{H}_y^r \\ \vec{H}_z^r \end{pmatrix} = \\ &= \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{c}_{\vec{k},\perp}^+ \cos\theta_i \\ \vec{c}_{\vec{k},\parallel}^+ \\ \vec{c}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{i\tau_r} + \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\vec{c}_{\vec{k},\perp}^+ \cos\theta_i \\ \vec{c}_{\vec{k},\parallel}^+ \\ \vec{c}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{-i\tau_r}\end{aligned}\quad (128)$$

and

$$\begin{aligned}\vec{H}_{0,t} &= \begin{pmatrix} \vec{H}_x^t \\ \vec{H}_y^t \\ \vec{H}_z^t \end{pmatrix} = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{b}_{\vec{k},\perp}^+ \cos\theta_i \\ \sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{b}_{\vec{k},\parallel}^+ \\ \vec{b}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{i\tau_t} + \\ &+ \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{b}_{\vec{k},\perp}^+ \cos\theta_i \\ \sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{b}_{\vec{k},\parallel}^+ \\ \vec{b}_{\vec{k},\perp}^+ \sin\theta_i \end{pmatrix} e^{-i\tau_t}\end{aligned}\quad (129)$$

where

$$\tau_i = \vec{k} \cdot \vec{r} + \omega_{\vec{k}} t = \omega_{\vec{k}} \left(\frac{\vec{s}_i \cdot \vec{r}}{c} + t \right)$$

$$\tau_r = \vec{k} \cdot \vec{r} + \omega_{\vec{k}} t = \omega_{\vec{k}} \left(\frac{\vec{s}_r \cdot \vec{r}}{c} + t \right)$$

and

$$\tau_t = \vec{k} \cdot \vec{r} + \omega_{\vec{k}} t = \omega_{\vec{k}} \left(\frac{\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})} \vec{s}_t \cdot \vec{r}}{c} + t \right)$$

are, respectively, the arguments of the incident, reflected and refracted Light Particles in the space wave vector \vec{k} and frequency $\omega_{\vec{k}}$, where $\omega_{\vec{k}} = \frac{kc}{\sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})}}$ (88).

Due to Huygens's principle, every point in direction of the Light represents as the source of second waves, therefore, in point of coordinate

$\vec{r} = (x, y, 0)$ on the threshold plane $z = 0$ at interface between air and metal in the coordinate system XYZ , the arguments of wave functions are the same $\tau_i = \tau_r = \tau_t$ [22]. Consequently, the laws of refraction and reflection on the boundary air-metal, takes the form:

$$\sin\theta_t = \frac{\sin\theta_i}{\hat{n}(\vec{k}, \omega_{\vec{k}})} \quad (130)$$

and

$$\theta_t = \pi - \theta_r \quad (131)$$

where θ_i , θ_r and θ_t are, respectively, the angles of incident, reflection and refraction; $\hat{n} = \sqrt{\varepsilon(\vec{k}, \omega_{\vec{k}})}$.

Considering the vector Bose-operators $\vec{a}_{\vec{k}}$, $\vec{c}_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ are, respectively, expanded to components of the normal $\vec{a}_{\vec{k},\perp}$, $\vec{c}_{\vec{k},\perp}$, $\vec{b}_{\vec{k},\perp}$ and parallel to the plane of incidence $\vec{a}_{\vec{k},\parallel}$, $\vec{c}_{\vec{k},\parallel}$, $\vec{b}_{\vec{k},\parallel}$, then, amplitudes of the wave are determined by the boundary conditions requiring that the tangential components of vectors electric and magnetic fields must be the same on both sides of interface air-metal:

$$\left. \begin{aligned} \vec{E}_x^i + \vec{E}_x^r &= \vec{E}_x^t \\ \vec{E}_y^i + \vec{E}_y^r &= \vec{E}_y^t \\ \vec{H}_x^i + \vec{H}_x^r &= \vec{H}_x^t \\ \vec{H}_y^i + \vec{H}_y^r &= \vec{H}_y^t \end{aligned} \right\} \quad (132)$$

These boundary conditions lead to the relationships between operators $\vec{c}_{\vec{k},\perp}$, $\vec{b}_{\vec{k},\perp}$ and $\vec{c}_{\vec{k},\parallel}$, $\vec{b}_{\vec{k},\parallel}$ via $\vec{a}_{\vec{k},\perp}$ and $\vec{a}_{\vec{k},\parallel}$. Indeed, letting that the vector incidence electric field $\vec{E}_{0,i}$ has an angle α_i with the plane of incidence, then we have

$$\left. \begin{aligned} \vec{a}_{\vec{k},\perp} &= \vec{a}_{\vec{k}} \sin\alpha_i \\ \vec{a}_{\vec{k},\parallel} &= \vec{a}_{\vec{k}} \cos\alpha_i \end{aligned} \right\} \quad (133)$$

which allows us to find

$$\left. \begin{aligned} \vec{c}_{\vec{k},\perp} &= r_{\perp} \vec{a}_{\vec{k},\perp} = r_{\perp} \vec{a}_{\vec{k}} \sin\alpha_i \\ \vec{c}_{\vec{k},\parallel} &= r_{\parallel} \vec{a}_{\vec{k},\parallel} = r_{\parallel} \vec{a}_{\vec{k}} \cos\alpha_i \\ \vec{c}_{\vec{k},\perp}^+ &= r_{\perp}^* \vec{a}_{\vec{k},\perp}^+ = r_{\perp}^* \vec{a}_{\vec{k}}^+ \sin\alpha_i \\ \vec{c}_{\vec{k},\parallel}^+ &= r_{\parallel}^* \vec{a}_{\vec{k},\parallel}^+ = r_{\parallel}^* \vec{a}_{\vec{k}}^+ \cos\alpha_i \end{aligned} \right\} \quad (134)$$

and

$$\left. \begin{aligned} \vec{b}_{\vec{k},\perp} &= t_{\perp} \vec{a}_{\vec{k},\perp} = t_{\perp} \vec{a}_{\vec{k}} \sin \alpha_i \\ \vec{b}_{\vec{k},\parallel} &= t_{\parallel} \vec{a}_{\vec{k},\parallel} = t_{\parallel} \vec{a}_{\vec{k}} \cos \alpha_i \\ \vec{b}_{\vec{k},\perp}^+ &= t_{\perp}^* \vec{a}_{\vec{k},\perp}^+ = t_{\perp}^* \vec{a}_{\vec{k}}^+ \sin \alpha_i \\ \vec{b}_{\vec{k},\parallel}^+ &= t_{\parallel}^* \vec{a}_{\vec{k},\parallel}^+ = t_{\parallel}^* \vec{a}_{\vec{k}}^+ \cos \alpha_i \end{aligned} \right\} \quad (135)$$

where quantities r_{\perp} , r_{\parallel} and t_{\perp} , t_{\parallel} are the amplitudes of reflection and refraction coefficients of normal and parallel to the plane of incidence, respectively, which present the Fresnel's equations:

$$r_{\perp} = \frac{\cos \theta_i - \hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_t}{\cos \theta_i + \hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_t} \quad (136)$$

$$r_{\parallel} = \frac{\hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_i - \cos \theta_t}{\hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_i + \cos \theta_t} \quad (137)$$

and

$$t_{\perp} = \frac{2 \cos \theta_i}{\cos \theta_i + \hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_t} \quad (138)$$

$$t_{\parallel} = \frac{2 \cos \theta_i}{\hat{n}(\vec{k}, \omega_{\vec{k}}) \cos \theta_i + \cos \theta_t} \quad (139)$$

Now, we try to treat the Hamiltonian operators of incident $\hat{H}_{R,i}$, reflected $\hat{H}_{R,r}$ and refracted $\hat{H}_{R,t}$ waves. In this respect, we separate the Hamiltonian operators. So, we may rewrite the expanded forms of the Hamiltonian operators of incident $\hat{H}_{R,i}$ radiation by following form:

$$\hat{H}_{R,i} = \hat{H}_{i,\parallel} + \hat{H}_{i,\perp} \quad (140)$$

where

$$\begin{aligned} \hat{H}_{i,\parallel} &= mc^2 \vec{a}_{0,\parallel}^+ \vec{a}_{0,\parallel} + \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k},\parallel}^+ \vec{a}_{\vec{k},\parallel} - \\ &- \frac{1}{2} \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{a}_{\vec{k},\parallel}^+ \vec{a}_{-\vec{k},\parallel}^+ + \vec{a}_{-\vec{k},\parallel} \vec{a}_{\vec{k},\parallel} \right) + \\ &+ \sum_{0 < k' < k_0} \left(\frac{\hbar^2 k'^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k}',\parallel}^+ \vec{a}_{\vec{k}',\parallel} + \\ &+ \frac{1}{2} \sum_{0 < k' < k'_0} \left(\frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{a}_{\vec{k}',\parallel}^+ \vec{a}_{-\vec{k}',\parallel}^+ + \vec{a}_{-\vec{k}',\parallel} \vec{a}_{\vec{k}',\parallel} \right) \end{aligned} \quad (141)$$

and

$$\begin{aligned}
\hat{H}_{i,\perp} = & mc^2 \vec{a}_{0,\perp}^+ \vec{a}_{0,\perp} + \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k},\perp}^+ \vec{a}_{\vec{k},\perp} - \\
& - \frac{1}{2} \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{a}_{\vec{k},\perp}^+ \vec{a}_{-\vec{k},\perp}^+ + \vec{a}_{-\vec{k},\perp} \vec{a}_{\vec{k},\perp} \right) + \\
& + \sum_{0 < k' < k_0} \left(\frac{\hbar^2 k'^2}{2m} + \frac{mc^2}{2} \right) \vec{a}_{\vec{k}',\perp}^+ \vec{a}_{\vec{k}',\perp} + \\
& + \frac{1}{2} \sum_{0 < k' < k'_0} \left(\frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{a}_{\vec{k}',\perp}^+ \vec{a}_{-\vec{k}',\perp}^+ + \vec{a}_{-\vec{k}',\perp} \vec{a}_{\vec{k}',\perp} \right)
\end{aligned} \tag{142}$$

where taking into account (133) with transformation (32), we get

$$\hat{H}_{R,i} = \hat{H}_{0,i} + 2 \sum_{0 < k < k_0} \chi_{\vec{k}} \vec{i}_{\vec{k}}^+ \vec{i}_{\vec{k}} \tag{143}$$

where

$$\hat{H}_{0,i} = mc^2 N_0 \tag{144}$$

is the energy of Light-Particles of incidence Light in the condensate;

$$\chi_{\vec{k}} = \hbar k c \tag{145}$$

is energy of incoming photons; $\vec{i}_{\vec{k}}^+$ and $\vec{i}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation of photons.

In analogy manner, we may rewrite the Hamiltonian operator of reflected $\hat{H}_{R,r}$ radiation:

$$\hat{H}_{R,r} = \hat{H}_{r,\parallel} + \hat{H}_{r,\perp} \tag{146}$$

where

$$\begin{aligned}
\hat{H}_{r,\parallel} = & mc^2 \vec{c}_{0,\parallel}^+ \vec{c}_{0,\parallel} + \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \right) \vec{c}_{\vec{k},\parallel}^+ \vec{c}_{\vec{k},\parallel} - \\
& - \frac{1}{2} \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{c}_{\vec{k},\parallel}^+ \vec{c}_{-\vec{k},\parallel}^+ + \vec{c}_{-\vec{k},\parallel} \vec{c}_{\vec{k},\parallel} \right) + \\
& + \sum_{0 < k' < k_0} \left(\frac{\hbar^2 k'^2}{2m} + \frac{mc^2}{2} \right) \vec{c}_{\vec{k}',\parallel}^+ \vec{c}_{\vec{k}',\parallel} + \\
& + \frac{1}{2} \sum_{0 < k' < k'_0} \left(\frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{c}_{\vec{k}',\parallel}^+ \vec{c}_{-\vec{k}',\parallel}^+ + \vec{c}_{-\vec{k}',\parallel} \vec{c}_{\vec{k}',\parallel} \right)
\end{aligned} \tag{147}$$

and

$$\begin{aligned}
\hat{H}_{r,\perp} = & mc^2 \vec{c}_{0,\perp}^+ \vec{c}_{0,\perp} + \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} \right) \vec{c}_{\vec{k},\perp}^+ \vec{c}_{\vec{k},\perp} - \\
& - \frac{1}{2} \sum_{0 < k < k_0} \left(\frac{\hbar^2 k^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{c}_{\vec{k},\perp}^+ \vec{c}_{-\vec{k},\perp}^+ + \vec{c}_{-\vec{k},\perp} \vec{c}_{\vec{k},\perp} \right) + \\
& + \sum_{0 < k' < k_0} \left(\frac{\hbar^2 k'^2}{2m} + \frac{mc^2}{2} \right) \vec{c}_{\vec{k}',\perp}^+ \vec{c}_{\vec{k}',\perp} + \\
& + \frac{1}{2} \sum_{0 < k' < k'_0} \left(\frac{\hbar^2 k'^2}{2m} - \frac{mc^2}{2} \right) \left(\vec{c}_{\vec{k}',\perp}^+ \vec{c}_{-\vec{k}',\perp}^+ + \vec{c}_{-\vec{k}',\perp} \vec{c}_{\vec{k}',\perp} \right)
\end{aligned} \tag{148}$$

where taking into account (134) with introducing new sort transformations

$$\left. \begin{aligned} r_{\perp} \vec{a}_{\vec{k}} &= \frac{r_{\perp} \vec{r}_{\vec{k}} + M_{\vec{k},\perp} r_{\perp}^* \vec{r}_{-\vec{k}}^+}{\sqrt{1 - M_{\vec{k},\perp}^2}} \\ r_{\parallel} \vec{a}_{\vec{k}} &= \frac{r_{\parallel} \vec{r}_{\vec{k}} + M_{\vec{k},\parallel} r_{\parallel}^* \vec{r}_{-\vec{k}}^+}{\sqrt{1 - M_{\vec{k},\parallel}^2}} \end{aligned} \right\} \tag{149}$$

where $M_{\vec{k},\perp}$ and $M_{\vec{k},\parallel}$ are the real symmetrical functions of a wave vector \vec{k} . Thus, the Hamiltonian operator of reflected $\hat{H}_{R,r}$ radiation takes diagonal form:

$$\hat{H}_{R,r} = \hat{H}_{0,r} + \sum_{0 < k < k_0} \xi_{\vec{k}} \vec{r}_{\vec{k}}^+ \vec{r}_{\vec{k}} \tag{150}$$

where

$$\hat{H}_{0,r} = mc^2 N_0 R(\vec{k} = 0, \omega_{\vec{k}=0}) \tag{151}$$

is the energy of reflected Light-Particles in the condensate;

$$\xi_{\vec{k}} = \hbar k c R(\vec{k}, \omega_{\vec{k}}) \tag{152}$$

is the energy of reflected polaritons; the reflection coefficient is

$$R(\vec{k}, \omega_{\vec{k}}) = |r_{\parallel}|^2 \cos^2 \alpha_i + |r_{\perp}|^2 (\vec{k}, \omega_{\vec{k}}) \tag{153}$$

$\vec{r}_{\vec{k}}^+$ and $\vec{r}_{\vec{k}}^-$ are, respectively, the Bose vector-operators of creation and annihilation of reflected polaritons.

By support of above prescription, it is easy to get the Hamiltonian operator of refracted $\hat{H}_{R,t}$ radiation which is:

$$\hat{H}_{R,t} = \hat{H}_{0,t} + 2 \sum_{0 < k < k_0} \eta_{\vec{k}} \vec{t}_{\vec{k}}^+ \vec{t}_{\vec{k}} \tag{154}$$

where

$$\hat{H}_{0,t} = mc^2 N_0 \hat{n}(\vec{k} = 0, \omega_{\vec{k}=0}) T(\vec{k} = 0, \omega_{\vec{k}=0}) \tag{155}$$

is the energy of refracted Light-Particles in the condensate;

$$\eta_{\vec{k}} = \hbar k c \hat{n} T(\vec{k}, \omega_{\vec{k}}) \quad (156)$$

is the energy of refracted polaritons; the refracted coefficient is

$$T(\vec{k}, \omega_{\vec{k}}) = |t_{||}|^2 \cos^2 \alpha_i + |t_{\perp}|^2 (\vec{k}, \omega_{\vec{k}}) \quad (157)$$

\vec{t}_k^+ , and \vec{t}_k^- are, respectively, the Bose vector-operators of creation and annihilation of refracted polaritons.

In these terms, the intensities of incident, reflection and refraction waves take the forms:

$$I_i = 2c \hat{H}_{R,i} \cos \theta_i \quad (158)$$

$$I_r = 2c \hat{H}_{R,r} \cos \theta_i \quad (159)$$

and

$$I_t = 2c \hat{H}_{R,t} \cos \theta_t \quad (160)$$

Hence we note that from the law conservation energy:

$$R + G = 1 \quad (161)$$

where the ratio $R = \frac{I_r}{I_i}$ is the reflection capability; the ratio $G = \frac{I_t}{I_i}$ is the transmission capability.

As result our investigation, the energies of incidence $\hat{H}_{0,i}$ (144), reflected $\hat{H}_{0,r}$ (151) and refracted $\hat{H}_{0,t}$ (155) Light-Particles in the condensate depend on the density of Light Particles in the condensate $\frac{N_0}{V}$ in air. Therefore, it needs to treat the BEC of Light Bosons in vacuum

VIII. BEC of Light Bosons in vacuum.

The connection between the ideal Bose gas and superfluidity in helium ^4He was first made by London [2] in 1938. He postulated that the ideal Bose gas undergoes a phase transition at sufficiently low temperatures to a condition in which the zero-momentum quantum state is occupied by a finite fraction of the atoms $\frac{N_0}{N}$ of liquid ^4He . This momentum-condensed phase was postulated by London for presentation of the superfluid component of ^4He . To be refused a broken of the Bose-symmetry law for the bosons being into condensate, we apply the Penrose-Onsager's definition of the Bose condensation [31]:

$$\lim_{N_0, N \rightarrow \infty} \frac{N_0}{N} = \text{const} \quad (162)$$

Hence in analogy manner, we postulate that a finite fraction of $\frac{N_0}{N}$ of the Light Bosons are been in the zero-momentum quantum states. Therefore, we may separate a number N_0 of Light Particles into condensed state by letting

$$\hat{N}_0 + \sum_{0 < k < k_0} \vec{a}_k^+ \vec{a}_k = \hat{N} \quad (163)$$

In statistical equilibrium, the equation (163) takes a following form:

$$N_{0,T} + \sum_{0 < k < k_0} \overline{\vec{a}_k^+ \vec{a}_k} = N \quad (164)$$

To find the form $\overline{\vec{a}_k^+ \vec{a}_k}$, we use of linear transformation presented in (32):

$$\overline{\vec{a}_k^+ \vec{a}_k} = \frac{1 + L_p^2 \overline{i_k^+ i_k^+}}{1 - L_p^2} + \frac{L_k}{1 - L_k^2} \left(\overline{i_k^+ i_{-k}^+} + \overline{i_k^- i_{-k}^-} \right) + \frac{L_k^2}{1 - L_k^2}$$

where $\overline{i_k^+ i_k^+}$ is the average number of photons with the wave vector \vec{k} at temperature T , presented by (46).

By the theorem of the Bloch-De-Dominis, we have

$$\overline{i_k^+ i_{-k}^+} = \overline{i_k^- i_{-k}^-} = 0 \quad (165)$$

Consequently, the equation for the density of Light Particles in the condensate takes a following form:

$$\frac{N_{0,T}}{V} = \frac{N}{V} - \frac{1}{V} \sum_{0 < k < k_0} \frac{L_k^2}{1 - L_k^2} - \frac{1}{V} \sum_{0 < k < k_0} \frac{1 + L_k^2 \overline{i_k^+ i_k^+}}{1 - L_k^2} \quad (166)$$

where the real symmetrical function L_k from a wave vector \vec{k} which equals to

$$L_k^2 = \frac{\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} - \hbar kc}{\frac{\hbar^2 k^2}{2m} + \frac{mc^2}{2} + \hbar kc} \quad (167)$$

As we have been seen in above the plasmon-polariton and resonance effects are fulfilled near the condensate level $\vec{k} = 0$. This fact claims to estimate the density Light bosons $\frac{N_0}{V}$ in the condensate at approximation $\vec{k} \rightarrow 0$. This request leads to vanishing the sums in right side of Eq.(166), and in turn $\frac{N_0}{V} \approx \frac{N}{V}$, at a temperature T .

IX. Experimental confirmation of existence Light Particles.

Now we consider the following cases:

1. In the case of the plasmon-polariton excitations around the plasmon frequency, we have a refractive index $\hat{n}(\vec{k}, \omega_{k,p}) = \frac{kc}{\omega_p}$, at $\omega_k \sim \omega_p$. From

the law of refraction, the incident angle must be $\theta_i = 0$, from (130). In this case, we may consider $\theta_t = 0$ because condition $\theta_t = \frac{\pi}{2}$ is absent due to (130). Thus, at $\theta_i = 0$ and $\theta_t = 0$ with approximation $\vec{k} \rightarrow 0$, we substitute $\hat{n}(\vec{k}, \omega_{\vec{k}}) = \frac{kc}{\omega_p}$ into (136)-(139). Then,

$$r_{\perp} = \frac{1 - \frac{kc}{\omega_p}}{1 + \frac{kc}{\omega_p}} \approx 1 \quad (168)$$

$$r_{\parallel} = -\frac{1 - \frac{kc}{\omega_p}}{1 + \frac{kc}{\omega_p}} \approx -1 \quad (169)$$

and

$$t_{\perp} = \frac{2}{1 + \frac{kc}{\omega_p}} \approx 2 \quad (170)$$

$$t_{\parallel} = \frac{2}{\frac{kc}{\omega_p} + 1} \approx 2 \quad (171)$$

Obviously, the reflection and refraction coefficients are $R = 1$ and $T = 4$, therefore, the Hamiltonian operators of reflected $\hat{H}_{R,r}$ the plasmon-polariton excitations around the plasmon frequency ω_p take the form:

$$\hat{H}_{R,r} = \hat{H}_{0,r} + \sum_{0 < k < k_0} \xi_{\vec{k}} \vec{r}_{\vec{k}}^+ \vec{r}_{\vec{k}} \quad (172)$$

where

$$\hat{H}_{0,r} = mc^2 N \quad (173)$$

is the energy of reflected Light-Particles in the condensate;

$$\xi_{\vec{k}} = \hbar kc \quad (174)$$

is the energy of reflected plasmon-polariton excitations.

The Hamiltonian operator of refracted $\hat{H}_{R,t}$ plasmon-polariton excitations, around the plasmon frequency ω_p , takes the form:

$$\hat{H}_{R,t} = 2 \sum_{k < k_0} \eta_{\vec{k}} \vec{t}_{\vec{k}}^+ \vec{t}_{\vec{k}} \quad (175)$$

where the energy of refracted plasmon-polariton excitations

$$\eta_{\vec{k}} = \frac{\hbar^2 k^2}{2m_l} \quad (176)$$

with effective mass

$$m_l = \frac{\hbar \omega_p}{8c^2} \approx 10^{-6} m_e$$

As we see the incoming incidence light induces the Bose-quasiparticles with an effective mass $m_l \approx \cdot 10^{-6} m_e$ that is small in regard to one in metal with effective mass $m_p = \frac{\hbar \omega_p}{2c^2} \approx 5 \cdot 10^{-6} m_e$ (117).

2. In the case of the resonance-polariton excitations, the refractive index is $\hat{n}(\vec{k}, \omega_{\vec{k},r}) = \frac{2m_0 c}{\hbar k}$, around the resonance frequency $\omega_{\vec{k},r} = \frac{\hbar k^2}{2m_0}$, with the approximation $\vec{k} \rightarrow 0$. Then, we may consider two important cases: Case 1. $\theta_i = 0$ and $\theta_t = 0$; Case 2. $\theta_i \neq 0$, $\theta_i \neq \frac{\pi}{2}$ and $\theta_t = \frac{\pi}{2}$. Substituting $\hat{n}(\vec{k}, \omega_{\vec{k}}) = \frac{2m_0 c}{\hbar k}$ into (136)-(139), gives the result for case 1. $\theta_i = 0$ and $\theta_t = 0$,

$$r_{\perp} = \frac{1 - \frac{2m_0 c}{\hbar k}}{1 + \frac{2m_0 c}{\hbar k}} \approx 1 \quad (177)$$

$$r_{\parallel} = -\frac{\frac{2m_0 c}{\hbar k} - 1}{\frac{2m_0 c}{\hbar k} + 1} \approx -1 \quad (178)$$

and

$$t_{\perp} = \frac{2}{1 + \frac{2m_0 c}{\hbar k}} \approx \frac{\hbar k}{m_0 c} \quad (179)$$

$$t_{\parallel} = \frac{2}{\frac{2m_0 c}{\hbar k} + 1} \approx \frac{\hbar k}{m_0 c} \quad (180)$$

Obviously, the reflection coefficient is $R = 1$. Therefore, the Hamiltonian operators of reflected $\hat{H}_{R,r}$ resonance-polariton excitations, around the resonance frequency $\omega_{\vec{k},r}$, takes the form:

$$\hat{H}_{R,r} = \hat{H}_{0,r} + \sum_{0 < k < k_0} \xi_{\vec{k}} \vec{r}_{\vec{k}}^+ \vec{r}_{\vec{k}} \quad (181)$$

where

$$\hat{H}_{0,r} = mc^2 N \quad (182)$$

is the energy of reflected Light-Particles in the condensate;

$$\xi_{\vec{k}} = \hbar k c \quad (183)$$

is the energy of reflected resonance-polariton excitations.

The Hamiltonian operator of refracted $\hat{H}_{R,t}$ resonance-polariton excitations, around the resonance frequency $\omega_{\vec{k},r}$, takes the form:

$$\hat{H}_{R,t} = 2 \sum_{k < k_0} \eta_{\vec{k}} \vec{t}_{\vec{k}}^+ \vec{t}_{\vec{k}} \quad (184)$$

where the energy of the refracted resonance-polariton excitations appear as Bose-quasiparticles

$$\eta_{\vec{k}} = \frac{\hbar^2 k^2}{2m_r} \quad (185)$$

with effective mass

$$m_r = \frac{m_0}{4} = 0.5 \cdot m_e$$

As we see the case, when $\theta_i = 0$ and $\theta_t = 0$ cannot lead to presence of Light Particles in the condensate which is needed to explain the problem of the enhancement of intensities of Light experiments SERS and metal films.

Now, we consider an interesting case $\theta_i \neq 0$, $\theta_i \neq \frac{\pi}{2}$, $\theta_t = \frac{\pi}{2}$ and $\alpha_i = \frac{\pi}{2}$, when the vector of electric field \vec{E}_i of incident Light is perpendicular to the plane of incidence. Then, $R = 1$ but

$$t_{\perp} \approx 2 \quad (186)$$

Then, the quadrat of refracted coefficient is $T = 4$, and in turn, the Hamiltonian operator of refracted $\hat{H}_{R,t}$ resonance-polariton excitations, around the resonance frequency $\omega_{\vec{k},r}$, take the form:

$$\hat{H}_{R,t} = \hat{H}_{0,t} + 2 \sum_{0 < k < k_0} \eta_{\vec{k}} \vec{t}_{\vec{k}}^+ \vec{t}_{\vec{k}} \quad (187)$$

where the energy of refracted resonance-polariton excitations appear as Bose-quasiparticles:

$$\eta_{\vec{k}} = 8m_0 c^2 \quad (188)$$

and the energy of Light Particles in the condensate into metal

$$\hat{H}_{R,0} = \lim_{k \rightarrow 0} \frac{8m_0 m c^3 N}{\hbar k} \rightarrow \infty \quad (189)$$

This result is a very important because it shows the existence of a constant electric field near the surface of metal.

As we know, the energy of Light-Particles of incident Light in the condensate is $\hat{H}_{0,i} = m c^2 N$, which determines the constant electric field \vec{E}_i of incident light on the surface of metal, may be define as:

$$\frac{E_i^2}{8\pi} = \frac{m c^2 N}{V} \quad (190)$$

In this respect, the energy of transmitted Light Particles in the condensate, which are concentrated near the interface of surface metal, represents the energy of constant electric field \vec{E}_t of transmitted light by following formulae:

$$\frac{E_t^2}{8\pi} = \frac{8m_0 m c^3 N}{\hbar k V} \quad (191)$$

The ratio

$$\frac{E_t^2}{E_i^2} = \frac{8m_0c}{\hbar k} \quad (192)$$

This result confirms the result of experiments [8-10] where highly unusual transmission properties have been shown for metal films perforated with a periodic array of subwavelength holes, because the electric field is highly localized inside the grooves (around 300-1000 times larger than intensity of incoming optical light). On other hand, it seems the ratio $\frac{E_t^2}{E_i^2}$ does not depend on the mass of Light Particles m . However, at a maximum quantity of wave number $k = k_0 = \frac{mc}{\hbar}$, presented in (44), and a minimum quantity of the ratio $\frac{E_t}{E_i} = 300$, we may find the mass $m = \frac{16m_e}{9 \cdot 10^4} = 1.8 \cdot 10^{-4}m_e$ from Eq.(192) which confirms the existence of Light Particles. Thus, the presence of the Light Particles in the condensate provides the launching of the surface Frölich-Schafroth bosons on the surface metal holes, at a ratio $\frac{E_t}{E_i} = 300 - 1000$.

X. Conclusion.

In conclusion, we show the advantages of the resonance-polariton model in comparison with the plasmon-polariton model. So, 1. the model plasmon-polariton is analysed at $T = 0$ but the model resonance-polariton at any temperature T ; 2. the mass of the plasmon-polariton is $m_p \approx 10^{-6}m_e$ but the mass of the resonance-polariton $m_r \approx 0.5m_e$, which implies the resonance-polariton has a heavy mass relative to the mass of Light Particle. This is the reason to suggest that the model surface plasmon-polariton cannot explain the observation results of the experiments connected with optical transmission through metal films, where the electric field is highly localized [8-10] as well as SERS experiments [12,13]. Indeed, the so-called resonance effect can explain these experiments; 3. in the case of plasmon-polariton excitations, the transmitted Light cannot be propagated by direction along of surface metal, and therefore, there are not the Light Particles in the condensate into metal which is needed to explain the problem of the enhancement of optical property of metal surface but in the case of resonance-polariton excitations, it is a possible.

In fact, the resonance effect is novel because it does not depend on temperature T . Hence, we note that the resonance effect connected with the dielectric response of medium considering by Bose gas consisting of the Frölich-Schafroth charged singlet bosons with charge $e_0 = 2e$ and mass $m_0 = 2m_e$.

Acknowledgements.

We are particularly grateful to Professor Marshall Stoneham F R S (London Centre for Nanotechnology, and Department of Physics and Astronomy

University College London, Gower Street, London WC1E 6BT, UK) for help with the English.

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